4 Practice!

Example 4. Find the scalar projection and vector projection of $\vec{b} = \langle 1, 1, 2\rangle$ onto $\vec{a} = \langle -2, 3, 1\rangle$.

$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{-2+3+2}{\sqrt{4+9+1}} = \frac{3}{\sqrt{14}}$$

$$\text{proj}_{\vec{a}}\vec{b} = (\text{comp}_{\vec{a}}\vec{b})\frac{\vec{a}}{|\vec{a}|} = \frac{3}{\sqrt{14}}\,\frac{\langle -2,3,1\rangle}{\sqrt{14}} = \frac{3}{14}\langle -2,3,1\rangle = \left\langle -\frac{6}{14}, \frac{9}{14}, \frac{3}{14}\right\rangle$$

Example 5. Find a unit vector that is orthogonal to both $\langle 2, 0, -1\rangle$ and $\langle 0, 1, -1\rangle$.

Let $\vec{a} = \langle a_1, a_2, a_3\rangle$ be such a unit vector.

Then $\vec{a}$ must satisfy:

$$2a_1 - a_3 = 0 \qquad a_1 = \frac{a_3}{2}$$
$$\Rightarrow$$
$$a_2 - a_3 = 0 \qquad a_2 = a_3$$

Also, since $\vec{a}$ is a unit vector:

$$a_1^2 + a_2^2 + a_3^2 = 1$$
$$\Rightarrow \frac{1}{4}a_3^2 + a_3^2 + a_3^2 = 1$$
$$\Rightarrow \frac{9}{4}a_3^2 = 1 \quad \Rightarrow a_3^2 = \frac{4}{9}$$
$$\Rightarrow a_3 = \frac{2}{3} \text{ (ignore negative part)}$$

$$\Rightarrow \vec{a} = \left\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right\rangle$$

Example 6. Determine whether the given vectors are orthogonal, parallel, or neither:

a. $\vec{a} = \langle 4, 5, -2\rangle$, $\vec{b} = \langle 3, -1, 5\rangle$
b. $\vec{u} = 9\vec{i} - 6\vec{j} + 3\vec{k}$, $\vec{v} = -6\vec{i} + 4\vec{j} - 2\vec{k}$

a. $\vec{a}$ and $\vec{b}$ are not parallel, since they are not scalar multiples of each other

$\vec{a}\cdot\vec{b} = 12 - 5 - 10 = -3$

$\Rightarrow \vec{a}$ and $\vec{b}$ are not orthogonal

b. $\vec{u}$ and $\vec{v}$ are parallel, since $\vec{u} = -\frac{3}{2}\vec{v}$

$\vec{u}\cdot\vec{v}$ therefore are not orthogonal